# Universalism doesn't entail extensionalism

Roberto Loss
robertoloss.com

Forthcoming in *Analysis*. Please refer to the published version. DOI:10.1093/analys/anab034

**Abstract**. In the literature on mereology it is often accepted that mereological universalism entails extensionalism. More precisely, many accept that, if parthood is assumed to be a partial order (and, thus, the relevant theory of parthood is taken to be at least as strong as 'core mereology'), the thesis that every plurality of entities has a mereological fusion entails the thesis that different composite entities have different proper parts. Central to this idea is the principle known as 'Weak Supplementation' which many take to impose an important constraint on the relation of proper parthood. In this paper I argue that this claim is false as the principle that I will call 'Minimal Supplementation' appears to be capable of doing all the work done by Weak Supplementation but without entailing extensionalism if conjoined with universalism and core mereology.

## 1. Introduction

*Mereological universalism* is the thesis that every plurality of entities has a mereological fusion. *Extensionalism* is the thesis that different composite entities have different proper parts. *Core mereology* is the thesis that parthood is reflexive, anti-symmetric, and transitive.[1] There appears to be widespread agreement in the contemporary debate on mereology that mereological universalism and core mereology jointly entail extensionalism. The main argument behind this idea was firstly presented by Varzi (2009) and appears to be widely endorsed in the literature.[2] It can be summed up by means of the following three claims:

(Claim 1) There are some models that don't intuitively represent possible models of parthood (like, for instance, the models in which some entity possesses just one proper part).

(Claim 2) The only way to rule out all the relevant unintended models is to assume both core mereology and the principle known in the literature as 'Weak Supplementation' (according to which, if *x* is a proper part of *y*, some part of *y* is disjoint from *x*).

(Claim 3) Weak Supplementation, core mereology, and universalism jointly entail extensionalism.

Contrary to common lore, in this paper I will argue that universalism, core mereology and Claim 1 are actually jointly *compatible* with the negation of extensionalism. I will do so by rejecting Claim 2. As I will show, there is a mereological principle (which I will label 'Minimal Supplementation') which appears to be perfectly capable of ruling out the

---

[1] See Varzi (2019: §2.2). Casati and Varzi (1999: 36) call it 'Ground Mereology'.

[2] See, among others, Rea (2010: 494-6), Calosi (2020: 4771-2), Smid (2015: 171), and Cotnoir (2016: 127-9).

relevant unintended models without entailing extensionalism if conjoined with core mereology. I will, thus, conclude that (*pace* Varzi 2009) universalism doesn't entail extensionalism.

## 2. The problem

I will take here the notion of *proper parthood* ('<') as primitive and define the notions of *parthood*, *overlap*, *disjointness* and *fusion* as follows (in what follows '*xx*', '*yy*', '*zz*' are plural variables and '$z \prec yy$' stands for '*z* is one of the *yy*'):

(Part) $x \leq y =_{df} x < y \vee x = y$

(Overlap) $Oxy =_{df} \exists z(z \leq x \wedge z \leq y)$

(Disjointness) $Dxy =_{df} \sim Oxy$

(Fusion) $xFyy =_{df} \forall z(z \prec yy \rightarrow z \leq x) \wedge \forall z\big(z \leq x \rightarrow \exists w(w \prec yy \wedge Owz)\big)$

Varzi (2009: 602-3) and Cotnoir (2016: 122-5) argue that the other definitions of the notion of mereological fusion that can be found in the literature (see Hovda 2009: 57-61 and Varzi 2019: §4.3) are problematic enough to be discarded, at least in this context. Be that as it may, given that Fusion is not only widely endorsed, but also appears to be pretty natural (by having its first conjunct requiring that the fusion 'contain' all the entities it fuses, and its second conjunct demanding that the entities fused completely 'cover' the fusion, so to speak; see Loss 2021: 5), what I will argue in what follows can be seen as also including the claim that non-extensionalist universalists don't need to reject Fusion and embrace some other definition of the notion of mereological fusion.

I will also assume that the following principles are true:

(Irreflexivity) $\forall x \sim (x < x)$

(Transitivity) $\forall x \forall y \forall z\big((x < y \wedge y < z) \rightarrow x < z\big)$

(Universalism) $\forall yy \exists x(xFyy)$

(notice that Irreflexivity, Transitivity, and Part entail that parthood is reflexive, anti-symmetric, and transitive, and thus that it complies with core mereology). For future reference, I will call this very minimal universalist mereology (which can be seen as simply adding Universalism to core mereology) 'CORU'.

Consider the models depicted in Figure 1. Intuitively, these models shouldn't be taken to be models of parthood (for some discussion see Cotnoir 2016: 125-7 and Varzi 2019: §3.1). However, it is easy to check that all the models of Figure 1 are models of CORU. Therefore, in order to exclude them, further mereological principles must be assumed. This is, however, where the problems for non-extensionalist universalists begin. Consider, as a matter fact, the following well-known principles:

(Weak Company) $\forall x \forall y \big(x < y \rightarrow \exists z (z < y \wedge z \neq x)\big)$

(Strong Company) $\forall x \forall y \big(x < y \rightarrow \exists z (z < y \wedge z \nleq x)\big)$

(Quasi Supplementation) $\forall x \forall y \big(x < y \rightarrow \exists z \exists w (z \leq y \wedge w \leq y \wedge Dzw)\big)$[3]

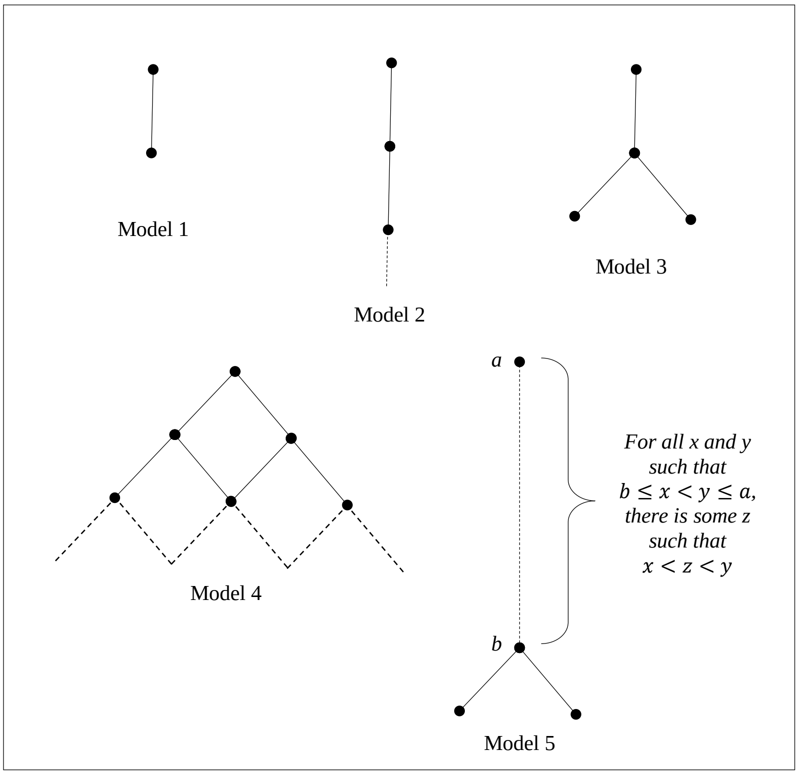


*Figure 1*

According to Weak Company, every composite entity must have more than just one proper part. According to Strong Company, for every proper part of an object there must be at least a second proper part that is not part of the first. According to Quasi Supplementation, every composite object must have at least two disjoint parts. Weak Company is successful only against the first model. Strong Company excludes only the first, the second, and the third model. Quasi Supplementation excludes only the first, the second, and the fourth. None of these principles excludes the fifth model (featuring a *dense* chain of proper parthood going from *b* to *a*). For this reason, it may seem that the only way to rule out these models is to accept Weak Supplementation, according to which if *x* is a proper part of *y*, then some part *z* of *y* is disjoint from *x*:

(Weak Supplementation) $\forall x \forall y \big(x < y \rightarrow \exists z (z \leq y \wedge Dzx)\big)$

---

[3] See Varzi (2019: section 3.1). Quasi Supplementation is defended as an alternative to Weak Supplementation by Gilmore (2014).

Indeed, some authors have gone as far as to claim that Weak Supplementation is 'constitutive of the meaning of 'proper part'' (Simons 1987: 116)[4] and that it 'expresses a minimal requirement which any relation must satisfy (besides reflexivity, anti-symmetry and transitivity) if it is to qualify as parthood at all' (Varzi 2008: 110-1). For this reason, the theory resulting from adding Weak Supplementation to core mereology has been labelled 'minimal mereology' (Casati and Varzi 1999: 39; Varzi 2019: §3.1). However, once Weak Supplementation is accepted, the resulting mereology becomes *in*compatible with the idea that two different composite entities can have the same proper parts. Consider, for instance, $b_1$ and $b_2$ in Figure 2. It follows from universalism that they must have a fusion. Varzi's (2009: 600) argument clearly shows that any fusion of $b_1$ and $b_2$ must contain both of them as proper parts, like the entity labelled '$a$' in Figure 2. But models like the one

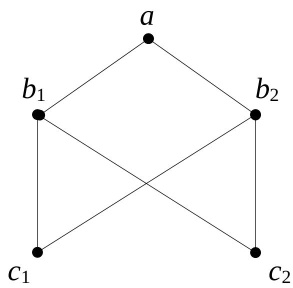


*Figure 2*

depicted in Figure 2 are clearly in violation of Weak Supplementation. Each of the *b*s is a proper part of $a$ and yet no part of $a$ is disjoint from any of the *b*s. Therefore, it may indeed seem that, assuming core mereology and Claim 1, mereological universalism is incompatible with the rejection of extensionalism.

## 3. The solution

Let two entities be *incomparable* just in case neither is a part of the other:

(Incomparable) $\mathbb{I}xy =_{df} x \nleq y \wedge y \nleq x$

Consider, then, the following principle, which I will label 'Minimal Supplementation':

(Minimal Supplementation)
$$\forall x \forall y \left(x < y \rightarrow \exists z\big(z \leq y \wedge \mathbb{I}xz \wedge \exists w \exists u(w \leq x \wedge u \leq z \wedge Dwu)\big)\right)$$

According to Minimal Supplementation, if an entity $x$ is a proper part of an entity $y$, then there is some part $z$ of $y$ that is both incomparable to $x$ and such that some of its parts is disjoint from some part of $x$. The addition of Minimal Supplementation to CORU is sufficient to rule out all the problematic models of Figure 1. In order to appreciate this

[4] See Cotnoir (2018) for some recent discussion of this idea.

point, consider the principle (which I will call ‘Full Company’) which says that if $x$ is a proper part of $y$, then there is some part of $y$ that is incomparable to $x$:

(Full Company) $\forall x \forall y(x < y \rightarrow \exists z(z \leq y \wedge \mathbb{I}xz))$

Full Company is clearly stronger than Strong Company. Taken together, Full Company and Quasi Supplementation rule out all the problematic models of Figure 1. In fact, the only model in Figure 1 that is a model of both Strong Company and Quasi Supplementation is model 5 which is clearly not a model of Full Company, as each of the *b*s is ‘comparable’ to all of the parts of $a$. Under the assumption of CORU, Full Company and Quasi Supplementation are jointly equivalent to Minimal Supplementation. As a matter of fact, (assuming CORU) not only does Minimal Supplementation clearly entail both Full Company and Quasi Supplementation, but it can also be easily proven that Full Company and Quasi Supplementation jointly entail Minimal Supplementation:

> Suppose that $b$ is a proper part of $a$. By Full Company, $a$ has a proper part $c$ that is incomparable to $b$, as required by the first two conjuncts of the consequent of Minimal Supplementation. The third conjunct says that some part of $b$ is disjoint from some part of $c$. Since $b$ and $c$ are incomparable, if either of them is atomic they clearly must be disjoint. Instead, if they are composite entities, it follows from Quasi Supplementation that they both have disjoint parts. Let $b_1$ and $b_2$ be two disjoint parts of $b$ and $c_1$ and $c_2$ be two disjoint parts of $c$. Consider $b_1$ and $c_1$. Either they are disjoint or they overlap. If they are disjoint, there is indeed a part of $b$ (namely, $b_1$) that is disjoint from a part of $c$ (namely, $c_1$). Suppose, instead, that they have a part $f$ in common. Since $f$ is a part of $c_1$ (which we are supposing to be disjoint from $c_2$) it must be disjoint from $c_2$. Therefore, we also have in this case that there is a part of $b$ (namely, $f$) that is disjoint from a part of $c$ (namely, $c_2$). Q.E.D.

It can be easily checked that the mereology resulting by adding Minimal Supplementation to CORU doesn’t entail extensionalism. Consider the model of Figure 2. $b_1$ is a proper part of $a$. $b_2$ is incomparable to $b_1$ and such that some of its parts (for instance, $c_1$) is disjoint from some part of $b_1$ (in this case, $c_2$; the same line of reasoning applies to $b_2$). $c_1$ is a proper part of both $b_1$, $b_2$ and $a$. $c_2$ is (i) part of $b_1$, $b_2$, and $a$, (ii) incomparable to $c_1$, and (iii) such that some of its parts (namely, $c_2$ itself) is disjoint from some part of $c_1$ (namely, $c_1$ itself; the same line of reasoning applies to $c_2$). Therefore, in the non-extensional model of Figure 2 both the *b*s and the *c*s comply with Minimal Supplementation.

It is well-known that if parthood is defined as in Part (see section 2), classical mereology can be axiomatized by means of Transitivity, Weak Supplementation, and Universalism:

*Classical Mereology*:

(Transitivity) $\forall x \forall y \forall z((x < y \wedge y < z) \rightarrow x < z)$

(Weak Supplementation) $\forall x \forall y(x < y \rightarrow \exists z(z \leq y \wedge Dzx))$

(Universalism) $\forall yy \exists x (xFyy)$

(see Hovda 2009: 81). The non-extensional and universalist mereology presented in this section (which may be labelled 'UNEM') can be axiomatized in a similar fashion:

*UNEM*:

(Transitivity) $\forall x \forall y \forall z ((x < y \wedge y < z) \rightarrow x < z)$

(Minimal Supplementation)
$\forall x \forall y \left( x < y \rightarrow \exists z (z \leq y \wedge \mathbb{I}xz \wedge \exists w \exists u (w \leq x \wedge u \leq z \wedge Dwu)) \right)$

(Universalism) $\forall yy \exists x (xFyy)$[5]

## 4. A stronger non-extensional and universalist mereology

UNEM is not the strongest non-extensional and universalist mereology complying with Transitivity, Minimal Supplementation and Universalism. Consider, as a matter of fact, the principle (which we may label 'Non-Extensional Supplementation' or 'NE-Supplementation' for short) according to which if $x$ is not part of $y$, then some part of $x$ is either disjoint from $y$ or it has the same proper parts of $y$ despite being different from $y$:

(NE-Supplementation)
$\forall x \forall y \left( x \nleq y \rightarrow \exists z (z \leq x \wedge \left( Dzy \vee (z \neq y \wedge \forall w (w < z \leftrightarrow w < y)) \right) \right)$

NE-Supplementation may be seen as a weaker version of the well-known Strong Supplementation principle (a theorem of both classical and extensional mereology):

(Strong Supplementation) $\forall x \forall y (x \nleq y \rightarrow \exists z (z \leq x \wedge Dzy))$

Whereas Strong Supplementation demands that if $x$ is not a part of $y$, then some part of $x$ must be disjoint from $y$, NE-Supplementation allows $y$ to overlap every part of $x$, provided that $x$ has a part that is different from $y$ but has the same proper parts of $y$ (thus leaving open the possibility of non-extensional scenarios).

[5] Transitivity, Minimal Supplementation, and Part entail that proper parthood is asymmetric (if $x$ is a proper part of $y$, Minimal Supplementation entails that there is some part of $y$ that is incomparable to $x$; but if $y$ was a proper part of $x$, then, by Transitivity, no part of $y$ could be incomparable to $x$). It follows, then, from Asymmetry and Transitivity that proper parthood is also irreflexive. Q.E.D.

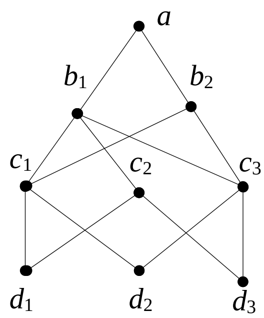


*Figure 3*

The model of Figure 3 is a model of UNEM[6] but *not* of NE-Supplementation: $c_2$ is not part of $b_2$, and yet (i) all the parts of $c_2$ overlap $b_2$ and (ii) there is no part of $c_2$ that has all and only the proper parts of $b_2$. The model of Figure 3 is also not a model of classical mereology, since it is not a model of Strong Supplementation ($c_2$ is not a part of $b_2$ and yet every part of $c_2$ overlaps $b_2$). The mereology resulting from the addition of NE-Supplementation to UNEM (which we may call 'Strong UNEM', or 'SUNEM' for short) is clearly compatible with the negation of extensionalism. In Figure 2, for instance, $a$ is not a part of $b_1$, but—despite the fact that every part of $a$ overlaps $b_1$ (contrary to what Strong Supplementation would require in this case)—there is a part of $a$ (namely, $b_2$) that has all and only the proper parts of $b_1$ (namely $c_1$ and $c_2$). Similarly, $b_2$ is not a part of $b_1$ and every part of $b_2$ overlaps $b_1$. However, there is a part of $b_2$ (namely, $b_2$ itself) that has all and only the proper parts of $b_1$. SUNEM is, therefore, a stronger non-extensional mereology than UNEM.

Interestingly, given SUNEM, Weak Supplementation can be shown to be *equivalent* to Extensionality of Proper Parthood:

(Extensionality of Proper Parthood)

$$\forall x \forall y \Big( \big( \exists z (z < x) \land \forall z (z < x \leftrightarrow z < y) \big) \rightarrow x = y \Big)$$

The fact that SUNEM and Weak Supplementation jointly entail Extensionality of Proper Parthood follows from the proof given by Varzi (2009: 600) and summarized above (recall that Transitivity and Universalism are theorems of SUNEM). Instead, the fact that, given SUNEM, Extensionality of Proper Parthood entails Weak Supplementation can be easily proved as follows:

---

[6] In Figure 3 each of the *c*s has the other *c*s as a 'incomparable supplements', so to say. Furthermore, for each pair of *c*s there is a part of the first that is disjoint from a part of the second. The same can be said of the *b*s. The *d*s are pairwise disjoint, so they clearly comply with Minimal Supplementation. To appreciate that every plurality of entities in Figure 3 has a fusion notice that without $b_2$ and $a$ Figure 3 is just a model of atomistic classical mereology in which $d_1$, $d_2$, and $d_3$ are the only existing atoms. Therefore, in Figure 3 $b_2$ is just an additional fusion of $c_1$ and $c_2$ taken together and $a$ is the fusion of $b_1$ and $b_2$ taken together (as required by universalism).

Suppose that $b$ is a proper part of $a$. By the asymmetry and irreflexivity of proper parthood it follows from Part that $a$ is not part of $b$. By NE-Supplementation we have that some part $z$ of $a$ is either (i) disjoint from $b$ or (ii) different from $b$ while having its same proper parts. In the second case, Extensionality of Proper Parthood entails that $b$ and $z$ have *no* proper parts so that, given Part, they must be disjoint. It follows, therefore, that in any case, some part of $a$ is disjoint from $b$. Q.E.D.

It is, thus, sufficient to add Extensionality of Proper Parthood to SUNEM to get classical mereology. Notice that, instead, the addition of Extensionality of Proper Parthood to UNEM doesn't result in a system that is as strong as classical mereology, as it is witnessed by the model of Figure 4 which is a model of both UNEM and Extensionality of Proper Parthood[7] but not of Weak Supplementation (since $b_3$ is a proper part of $a$ but there is no part of $a$ that is disjoint from $b_3$).[8]

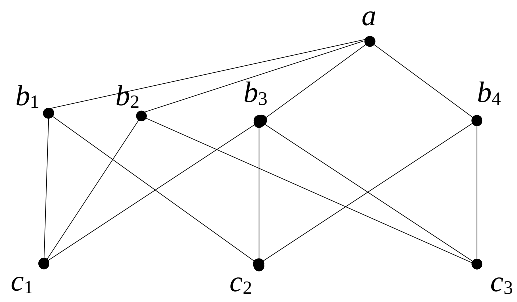


*Figure 4*

## 5. Conclusion

In this paper I have argued that contrary to what is often assumed in the literature, universalism and the negation of extensionalism are jointly compatible with core mereology and the idea that models like those in Figure 1 are not models of parthood. I did so by introducing a 'minimal' supplementation principle which appears to do all the work that Weak Supplementation is usually invoked for but without entailing extensionalism when coupled with core mereology and universalism. It can thus be concluded not only that (*pace* Varzi 2009) universalism doesn't entail extensionalism, but also that at least those who accept both core mereology and the idea that the models of Figure 1 are not models of

[7] In Figure 4 (i) each of the $b$s has the other $b$s as 'incomparable supplements', (ii) for each pair of $b$s there is a part of the first that is disjoint from a part of the second, and (iii) the $c$s are pairwise disjoint. (i)-(iii) guarantee that both the $b$s and the $c$s comply with Minimal Supplementation. In order to appreciate that every plurality of entities in Figure 4 has a fusion notice that without $b_3$ Figure 4 is just a model of atomistic classical mereology in which $c_1$, $c_2$, and $c_3$ are the only existing atoms. Therefore, in Figure 4 $b_3$ is just an additional fusion of the $c$s (while in classical mereology $a$ must be the *only* fusion of the $c$s).

[8] Notice that the model of Figure 4 is not a model of NE-Supplementation: $a$ is not part of $b_3$ and yet (i) every part of $a$ overlaps $b_3$ and (ii) no part of $a$ is different from $b_3$ and has the same proper parts of $b_3$.

parthood should regard the mereology obtained by extending core mereology with Minimal Supplementation as the truly 'minimal' mereology.[9]

## References


Casati, R. and A. Varzi. 1999. *Parts and Places: The Structures of Spatial Representation*. MIT Press.

Calosi, C. 2020. An elegant universe. *Synthese* 197: 4767-4782.

Cotnoir, A. 2016. Does universalism entail extensionalism? *Noûs* 50 (1):121-132.

Cotnoir, A. 2018. Is weak supplementation analytic? *Synthese*. https://doi.org/10.1007/s11229-018-02066-9.

Gilmore, C. 2014. Parts of propositions, in S. Kleinschmidt (ed.), *Mereology and Location*, Oxford: Oxford University Press: 156–208.

Hovda, P. 2009. What is classical mereology? *Journal of Philosophical Logic* 38(1): 55-82.

Loss R. 2021. Two notions of fusion and the landscape of extensionality. *Philosophical Studies*. https://doi.org/10.1007/s11098-021-01608-1.

Rea, M. 2010. Universalism and extensionalism. A reply to Varzi. *Analysis*, *70*, 490–496.

Simons, P. 1987. *Parts. A Study in Ontology*. Oxford: Clarendon.

Smid, J. 2015. A puzzle concerning boundaries, dependence, and parthood. *Analytic Philosophy* 56 (2):169-176.

Varzi, A. 2008. The extensionality of parthood and composition. *Philosophical Quarterly* 58(230): 108-133.

Varzi, A. 2009. Universalism entails extensionalism. *Analysis*, 69: 599–604.

Varzi, A. 2019. Mereology. In *The Stanford Encyclopedia of Philosophy*, Spring 2019 ed. E. N. Zalta. Stanford University <https://plato.stanford.edu/archives/spr2019/entries/mereology>.



[9] Many thanks to three anonymous referees for this journal for their helpful comments.