Mereological endurantism and being a whole at a time: reply to Costa

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[Penultimate draft. Forthcoming in *Inquiry*. doi: 10.1080/0020174X.2021.1990797]

Abstract. Damiano Costa has recently offered a novel mereological definition of endurantism based on the idea that for an object to be wholly present at a time is for it to be a whole at that time. In this paper I argue that Costa's is not a definition of endurantism, since the idea that every object is a whole at every time it exists can be accepted by endurantists and perdurantists alike.

Keywords Three-dimensionalism; wholly present; mereology; persistence; time

1. Introduction

The main slogan of the theory of persistence known as 'endurantism' is that entities persist in time by being 'wholly present' at every time they exist. However, the notion of *being wholly present* has proved to be very difficult to define by means of other notions.[1] As an effect of this, in recent years a 'locative turn' (Costa 2017) appears to have taken place in the debate on endurantism, so that many authors now take endurantism to be defined as the idea that entities persist in time by being *temporally multi-located*—in the sense of having multiple 'exact locations' at different times.[2] In particular, what we may call a *purely mereological* definition of endurantism has proved very difficult to come by (where by 'purely mereological' I mean here a definition such that the only non-logical notions it employs are mereological: 'part', 'proper part', 'overlap', 'mereological sum', *et cetera*).[3] The problems of mereological definitions of endurantism are nicely summed up by Sider (2001: 64):

> What is it for *x* to be 'wholly present' at *t*? The idea is presumably that every part of *x* exists at *t*. But every part at what time? For three-dimensionalists, the parthood relation is temporally relative, and so 'every part of *x* exists at *t*' is incomplete since 'part of' is temporally unqualified. We might take '*x* is wholly present at *t*' to mean that everything that is part of *x* at *t* exists at *t*. But then the claim that objects are always wholly present whenever they exist becomes utterly trivial, and not the controversial doctrine we thought it was, for no one would deny that a part of an object at a given time must exist then. […]
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> Another sense of 'wholly present' might be defined as follows:

[1] For an overview of this debate see, for instance, Crisp and Smith (2005).

[2] See, among others, Sattig (2006), Eagle (2016), Gilmore (2018), and Leonard (2018).

[3] So, for instance, a definition of endurantism employing a (non-mereologically defined) notion of *constitution*—as the alternative definitions that Costa (2020: 9) himself offers—doesn't count as purely mereological in this sense.

> x is *strongly wholly present* throughout interval T $=_{df}$ everything that is at *any* time in T part of x exists and is part of x at every time in T.
>
> But the claim that objects are always *strongly* wholly present throughout their careers is too strong a formulation of three-dimensionalism, for it entails the impossibility of gain or loss of parts. […But] mereological essentialism should not be *built into* the statement of three-dimensionalism, for most three-dimensionalists reject it.

Recently, Costa (2020) has offered a novel purely mereological definition of endurantism which he claims to be up to the task. The intuitive gloss on which Costa's definition is based is the following:

(IGW) Something is wholly present at a time if and only if it is present at that time by being a whole at that time.

Such a gloss is then turned into a more precise definition as follows:

(IGW'') x is wholly present at a time t if and only if x exists at t and if x is complex at t, then x is identical to a sum of some of its proper parts at time t.[4]

In this paper I will argue that—*pace* Costa—(IGW'') is *not* a definition of endurantism. As I will show, in fact, (IGW'') appears to be a perfectly sensible thesis that both endurantists and perdurantists can accept.

2. Not a definition of endurantism

Assuming in the background a B-theoretic, eternalist theory of time (according to which past, present, and future entities all exist and there is no 'privileged present' constantly changing as time 'goes by') it is natural for endurantists to employ as their primitive mereological notion the three-place notion of *parthood-at-a-time* ('$\leq_t$').[5] The temporal notions of proper-parthood-at-a-time ('$<_t$'), overlap-at-a-time ('O_t'), and sum-at-a-time ('S_t') can be defined in terms of the notion of parthood-at-a-time as follows (notice that, given the importance in this paper of clearly distinguishing between *temporal* and *atemporal* notions I will use phrases like 'x is part-at-t of y', 'x and y overlaps-at-t', *et cetera*, instead of the more colloquial 'x is part of y at t', 'x overlaps y at t', *et cetera*):

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[4] As it will become clear below, my criticism applies also to the other purely mereological definition of endurantism provided by Costa, namely those he calls 'Whole presence 1' (p. 6) and 'Whole presence 2' (pp. 7-8). Instead, the definitions he calls 'Whole presence for constitution theorists' (p. 9) and 'Whole presence for time traveller's (p. 10) are not *purely* mereological and are, thus, beyond the scope of this paper.

[5] Notice that in what follows I will take times to be *instants* rather than intervals of time.

(Proper-part-*t*) $x <_t y =_{df} x \leq_t y \wedge x \neq y$

x is a proper-part-at-*t* of *y* if and only if *x* is a part-at-*t* of *y* and *x* is different from *y*

(Overlap-*t*) $O_t xy =_{df} \exists z(z \leq_t x \wedge z \leq_t y)$

x overlaps-at-*t* *y* if and only if some *z* is a part-at-*t* of both *x* and *y*

(Sum-*t*) $S_t(x, \phi y) =_{df} \forall z\big(O_t zx \leftrightarrow \exists y(\phi y \wedge O_t yz)\big)$

x is a sum-at-*t* of every *y* such that ϕy if and only if something overlaps-at-*t* *x* if and only if it overlaps-at-*t* some *y* such that ϕy

Costa (2020: 6) employs a notion of parthood-at-a-time according to which an entity *x* can be a part-at-*t* of an entity *y* only provided that *x* is *entirely located* at *t*:

> I am here assuming a notion of temporary parthood whereby a temporary part is such that it is entirely located at the relevant time […] where an entire location of *x* is conceived of as a region which has an exact location of *x* as a part […]. To illustrate, under such an understanding, a perdurantist would say that the current temporal part of my left hand is a part now of me, whereas my left hand as a perduring entity is not a part now of me, for it is not entirely located at the present instant […]' (Costa 2020: 6)

Notice that this assumption isn't completely uncontroversial. For instance, philosophers embracing both 'mereological' and 'locational' perdurantism (Gilmore 2018: section 6.3.2) and defining the notion of part-at-a-time along the lines of Sider (2001: 57; see also below) will accept that in many cases something can be part-at-*t* of something else while being exactly and, thus, entirely located at a region of spacetime not contained by *t*. For instance, these philosophers will claim that, while there are many times *t* such that my right arm is part-at-*t* of my body, the exact location of my arm is a four-dimensional region of spacetime that is not contained as a part by any of these times, so that my right arm isn't entirely located at any of them. Be that as it may, however, in what follows I will simply follow Costa and accept this assumption.

Assuming that exact location is a function and so that objects are not multi-located (a claim Costa's theory should at least be compatible with), it follows from Costa's assumption concerning the notion of parthood-at-a-time that any entity that exists at *more* than one time can*not* be a part of itself at any time *t* at which it exists (in what follows '$E_t x$' stands for 'x exists at *t*', and 'Px' stands for 'x exists at more than one time': '$Px =_{df} \exists t \exists u(t \neq u \wedge E_t x \wedge E_u x)$'):

(PPT) $\forall x \forall t\big((Px \wedge E_t x) \rightarrow \sim x \leq_t x\big)$

In fact, if an entity x was part-at-t of itself in this sense, then it would have to be entirely located at *t*, and thus exactly located *only* at *t*. However, this would prevent it from existing at other times (given the highly plausible assumption that an object exists at a time if only if it is weakly located at that time, and thus, that an object exists at a time *t* if and only if its exact location overlaps *t*).[6]

Letting '$C_t x$' stand for 'x is a composite object at *t*' (namely, an object with proper parts-at-*t*) the notion of being wholly present at a time *t* is defined by Costa (2020: 6) along the following lines:

(WHP) x is wholly present at $t =_{df} E_t x \wedge C_t x \wedge \exists z(S_t(z, y <_t x) \wedge x = z)$

Therefore, Costa's mereological definition of endurantism can be formulated as follows:

(END) $\forall x \forall t\big((E_t x \wedge C_t x) \rightarrow \exists z(S_t(z, y <_t x) \wedge x = z)\big)$
For every object *x* and time *t*, if *x* exists at *t* and is a composite object at *t*, then *x* is identical to a sum-at-*t* of the proper parts-at-*t* of *x*

The problem with (END) is that also four-dimensionalists can accept that every entity that exists-at-*t* and has proper parts-at-*t* is identical to the sum-at-*t* of their proper-parts-at-*t*—and, thus, 'wholly present' at *t* in this sense. Consider, for instance, Sider's (2001) version of four-dimensionalism. Sider (2001) defines the notions of *instantaneous temporal part* and *parthood-at-a-time* in terms of an atemporal notion of parthood as follows:

(ITT) 'x is an instantaneous temporal part of y at instant $t =_{df}$ (1) *x* is a part of *y*; (2) *x* exists at, but only at, *t*; and (3) *x* overlaps every part of *y* that exists at *t*.' (Sider 2001: 59)

(P@T) 'x is part of y at t iff *x* and *y* each exist at *t*, and *x*'s instantaneous temporal part at *t* is part of *y*'s instantaneous temporal part at *t*.' (Sider 2001: 57)

In addition, Sider formulates four-dimensionalism as follows:

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[6] I am here relying on the following principles concerning the notions of exact, weak, and entire location:

(WL) *x* is weakly located at *r* if and only if there is a region *s* such that *x* is exactly located at *s* and *r* overlaps *s*

(ENL) *x* is entirely located at *r* if and only if there is a region *s* such that *x* is exactly located at *s* and *s* is part of *r*

(see Parsons 2007: 204-5).

(4D) ‘necessarily, each spatiotemporal object has a temporal part at every moment at which it exists.’ (Sider 2001: 59)

Given these assumptions, it is easy to prove (assuming that the atemporal notion of parthood is reflexive and transitive) that the notion of parthood-at-a-time defined in (P@T) is both ‘conditionally reflexive’ (as we may say) and transitive (to avoid ambiguities, I will use in what follows ‘$\leq_t^4$’ for the notion of parthood-at-a-time defined in (P@T)):

Conditional Reflexivity: $\forall x \forall t(E_t x \rightarrow x \leq_t^4 x)$

Proof. Suppose *x* exists at *t*. By (4D), there is an entity *y* that is an instantaneous temporal part of *x* at *t*. By the reflexivity of atemporal parthood, *y* is a part of *y*. This means that *x*’s instantaneous temporal part at *t* is part of *x*’s instantaneous temporal part at *t*. We have, then, from (P@T) that *x* is a part of *x* at *t*. Q.E.D.

Transitivity: $\forall x \forall y \forall z \forall t\big((x \leq_t^4 y \wedge y \leq_t^4 z) \rightarrow x \leq_t^4 z\big)$

Proof. Suppose *x* is a part of *y* at *t* and *y* is a part of *z* at *t*. We have from (P@T) that some entities *v*, *u* and *w* are such that (i) *v* is an instantaneous temporal part of *x* at *t*, *u* is an instantaneous temporal part of *y* at *t*, *w* is an instantaneous temporal part of *z* at *t*, and (ii) *v* is part of *u*, and *u* is part of *w*. It follows from the transitivity of atemporal parthood that *v* is part of *w*, which entails, by (P@T), that *x* is a part of *z* at *t*. Q.E.D.

Similarly, four-dimensionalists can define a notion behaving like Costa’s notion of part-at-a-time (for which I will use ‘$\leq_t^C$’) as follows (where ‘$@_<$’ stands for the notion of entire location):

(P@T$_C$-1) $x \leq_t^C y =_{df} x \leq_t^4 y \wedge x @_< t$

Notice that, given the principles concerning existence-at-a-time, entire and exact location we are assuming in the background,[7] (P@T$_C$-1) entails that *x* is a part-at-*t* of *y* in this sense if and only if *x* is a part-at-*t* of *y* in the sense of (P@T) and *x* exists only at *t*:

(P@T$_C$-2) $x \leq_t^C y \leftrightarrow \big(x \leq_t^4 y \wedge \forall u(E_u x \rightarrow u = t)\big)$

It is then straightforward to check that the notion defined in (P@T$_C$-1) obeys the following counterparts of Conditional Reflexivity and Transitivity:

[7] Namely: (i) exact location is unique; (ii) *x* exists at *t* if only if *x* is weakly located at *t*; (iii) *x* is weakly located at *t* if and only if *x*’s exact location overlaps *t*; (iv) *x* is entirely located at *t* if and only if *x*’s exact location is part of *t*.

Conditional Reflexivity*: $\forall x \forall t \left(\left(E_t x \wedge \forall u (E_u x \to u = t) \right) \to x \leq_t^C x \right)$

Transitivity*: $\forall x \forall y \forall z \forall t \left((x \leq_t^C y \wedge y \leq_t^C z) \to x \leq_t^C z \right)$

Suppose, then, that a certain entity *a* existing at more than one time exists at time *t* and has proper-parts-at-*t*:[8]

(1) $Pa \wedge E_t a \wedge \exists y (y <_t^C a)$

Given what we said thus far, it is thus possible to prove that *a* is a sum-at-*t* of its proper parts, and thus, given (WHP), that *a* is wholly present at *t*. Given the counterpart of (Sum-*t*) defined by means of '$\leq_t^C$', to say that *a* is a sum-at-*t* of its proper parts is to say that something overlaps-at-*t* *a* if and only if it overlaps-at-*t* some of its proper parts:

(Sum-*t**) $S_t(a, y <_t^C a) \leftrightarrow \forall z \left(O_t za \leftrightarrow \exists y (y <_t^C a \wedge O_t yz) \right)$

Therefore, in order to prove that *a* is a sum-at-*t* of its proper parts it is sufficient to prove both directions of the biconditional featuring on the right-hand-side of (Sum-*t**):

(Sum-*t**-rhs) $\forall z \left(O_t za \leftrightarrow \exists y (y <_t^C a \wedge O_t yz) \right)$

As for the left-to-right direction of (Sum-*t**-rhs), suppose that a certain entity *b* overlaps-at-*t* *a*:[9]

(2) $O_t^C ab$

This means that there is some entity *c* such that *c* is a part-at-*t* of both *a* and *b*

(3) $c \leq_t^C a \wedge c \leq_t^C b$

It follows from (PPT), the first conjunct of (3), and the first conjunct of (1) that *c* is a *proper*-part-at-*t* of *a*:

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[8] This notion of proper-part-at-*t* is defined as follows:

$$x <_t^C y =_{df} x \leq_t^C y \wedge x \neq y$$

[9] This notion of overlap-*t* is defined as follows:

$$O_t^C xy =_{df} \exists z (z \leq_t^C x \wedge z \leq_t^C y)$$

(4) $c <_t^C a$

From (P@T$_C$-2) we have that *c* exists only at *t*. It follows, thus, from Conditional Reflexivity* that *c* is part-at-*t* of itself. Therefore, *c* is a proper part of *a* that is both a part-at-*t* of itself and of *b*, which means that *b* overlaps-at-*t* something that is a proper-part-at-*t* of *a*:

(5) $\exists z(z <_t^C a \wedge O_t^C zb)$[10]

From (2) and (5) it follows, by generalization, that everything that overlaps-at-*t* *a* overlaps-at-*t* at least some of its proper-parts-at-*t*:

(6) $\forall z\left(O_t^C za \rightarrow \exists y(y <_t^C a \wedge O_t^C zy)\right)$

As for the right-to-left direction of (Sum-*t**-rhs), suppose that two entities *b* and *c* are such that *b* overlaps-at-*t* *c* and *c* is a proper-part-at-*t* of *a*:

(7) $O_t^C bc \wedge c <_t^C a$

b and *c* have thus a part-at-*t* in common. Let *d* be their common part-at-*t*:

(8) $d \leq_t^C b \wedge d \leq_t^C c$

d is a part-at-*t* of *c*. In turn, *c* is a (proper) part-at-*t* of *a* (as the second conjunct of (7) says). From Transitivity* it follows, thus, that *d* is also part-at-*t* of *a*:

(9) $d \leq_t^C a$

Therefore, *d* is a part-at-*t* of both *a* and *b*, so that they overlap-at-*t*:

(10) $O_t^C ab$

It follows from (7) and (10) (by existential generalization, conditional proof, and universal generalization) that everything that overlaps-at-*t* a proper part-at-*t* of *a* also overlaps-at-*t* *a*:

[10] Notice that (5) can be shown to follow from (3) even without assuming (PPT) and allowing, thus, persisting objects to be part of themselves at times at which they exist. In fact, if the entity *c* that we are supposing to be part-at-*t* of both *a* and *b* is identical to *a* it still follows that *b* overlaps-at-*t* some of the proper parts-at-*t* of *a*. We are assuming that *a* has proper parts-at-t. Let *d* any of these proper parts-at-t of *a*. By (P@T$_C$-2), *d* exists only at *t*. By Conditional Reflexivity* *d* is thus part-at-*t* of itself. However, by Transitivity* *d* is also a part-at-t of *b*, so that *d* is indeed a proper part-at-t of *a* that overlaps-at-t *b*.

(11) $\forall z(\exists y(y <_t^C a \wedge O_t^C yz) \rightarrow O_t^C za))$

(Sum-t*-rhs) clearly follows from (6) and (11), and from (Sum-t*-rhs) and (Sum-t*) we have, thus, that *a* is a sum-at-*t* of everything that is a proper-part-at-*t* of *a*:

(12) $S_t^C(a, y <_t^C a)$

Hence, there is something that is both (i) the sum-at-*t* of everything that is a proper-part-at-*t* of *a* and (ii) identical to *a*:

(13) $\exists z(S_t^C(z, y <_t a) \wedge z = a))$

Since we are assuming that *a* exists-at-*t* and *a* has proper-parts-at-*t* it follows from (WHP) that *a* is indeed wholly present at *t*:

(14) a is wholly present at *t*

Therefore, Costa's definition of 'being wholly present' entails that also four-dimensionalists like Sider (2001) should claim that every entity that exists at a time *t* and has proper parts at *t* (in Costa's sense) is wholly present at *t*.

The root of the problem with (WHP) appears to be following. In order to argue that his definition of being wholly present is able to distinguish between three-dimensionalism and four-dimensionalism, Costa claims:

> In general, a four-dimensionalist would not take a four-dimensional object to be identical to a *sum* of proper parts that an object has at any time of its persistence. At best, a four-dimensionalist would take such a *sum* to be one of the proper temporal parts of the object at the given time. But since such a *sum* is a proper temporal part of the object, it is also is a proper part of it, and therefore it is not numerically identical to it. (Costa, 2020: 11; italics mine).

In this passage Costa appears to equivocate between the *temporal* notion of sum and the *atemporal* one. In fact, when Costa says that 'a four-dimensionalist would not take a four-dimensional object to be identical to a *sum of proper parts* that an object has at any time of its persistence' this claim is *correct* if the notion of sum at play is the *atemporal* notion of sum (formulated by means of the *atemporal* notions of parthood and overlap):

(Sum) $S(x, \phi y) =_{df} \forall z\big(Ozx \leftrightarrow \exists y(\phi y \wedge Owz)\big)$

In fact, if we assume, as Costa does, that the relevant proper parts at *t* of an object *x* exist only at *t*, then the atemporal sum of those entities is indeed only an instantaneous temporal part of *x*. Instead, as we have proved above, the claim is *incorrect* if understood by means of the *temporal* notion of sum, or 'sum-at-*t*', as four-dimensionalists can agree that every entity that exists-at-*t* and has proper-parts-at-*t* (in Costa's non-standard sense) *is* indeed the sum-at-*t* of all of its proper parts-at-*t*. However, Costa's definition of endurantism crucially employs a *temporal* notion of sum. Therefore, the fact that four-dimensionalists don't take persisting objects to be identical to *atemporal* sums of proper parts they have at any time of their persistence is of no help in this case.[11]

3. Conclusion

Costa (2020) has proposed a novel purely mereological definition of endurantism. In this paper I have showed that the mereological definition of the notion of whole presence given by Costa appears to be insufficient to distinguish between four-dimensionalism and three-dimensionalism. Therefore, it seems possible to conclude that the prospects of a purely mereological definition of endurantism still look dim.

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[11] Notice, furthermore, that (no matter how they may define the atemporal notion of parthood in terms of their primitive temporal one) also Costa's endurantists appear to be committed to rejecting the idea that persisting objects are identical to *atemporal* sums of proper parts they have at any time of their persistence. Suppose, in fact, that *a* is a persisting object that exists both at time T1 and at time T2. Let the *bb* be the proper parts-at-T1 of *a* and the *cc* be the proper parts-at-T2 of *a*. Following Costa's notion of part-at-a-time (according to which if *x* is a part-at-t of *y*, then *x* exists only at *t*; see above), we have both that all of the *bb* and their parts exists only at T1 and that all of the *cc* and their parts exists only at T2. If *a* was an atemporal sum of both the *bb* and the *cc* it would follow from the definition of atemporal sum (see (Sum) above) that each of the *bb* overlaps some of the *cc* and that each of the *cc* overlaps some of the *bb*. This is, however, *impossible* given that every part of the *bb* exists only at T1 and every part of the *cc* exists only at T2.