

# Branching time, actuality and the puzzle of retrospective determinacy

Roberto Loss

robertoloss@gmail.com

Area

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## Abstract

The supervaluationist approach to branching time ('SBT-theory') appears to be threatened by the *puzzle of retrospective determinacy*: if yesterday I uttered the sentence 'It will be sunny tomorrow' and only in some worlds overlapping at the context of utterance it is sunny the next day, my utterance is to be assessed as neither true nor false even if today is indeed a sunny day. John MacFarlane (2008) has recently criticized a promising solution to this puzzle for falling short of an adequate account of 'actually'. In this paper I aim to rebut MacFarlane's criticism. To this effect, I will argue that: (i) 'actually' can be construed either as an *indexical* or as a *nonindexical* operator; (ii) if 'actually' is nonindexical, MacFarlane's criticism is invalid; (iii) there appear to be independent reasons for SBT-theorists to claim that 'actually' is a nonindexical expression.

## 1. *The puzzle of retrospective determinacy*

According to the *branching picture of time*, time is modelled by a tree of possible worlds overlapping towards the past and branching towards the future. Each 'node' on the

branching tree of time has, thus, a unique past but many possible futures, none of which 'marked' as *the* way things will turn out to be. For simplicity's sake, let us take each node to be a possible *context of utterance*. Consider then a context  $c_1$ , and suppose that the time of  $c_1$  is  $t_1$ . Suppose, furthermore, that: (i) the contexts  $c_2$  and  $c_3$  are the only two possible futures of  $c_1$  as to what might happen at  $t_2$ ; (ii) it is sunny at  $c_2$  and it is raining at  $c_3$ ; (iii) in  $c_1$  I utter the sentence:

- (1) It will be sunny at  $t_2$

According to the *indeterminacy intuition* (MacFarlane 2003, 2008), the following claim is correct:

- (2) The utterance  $u$  of (1) at  $c_1$  is neither true nor false

To vindicate this intuition, the branching theorist can resort to *supervaluationism*,<sup>1</sup> according to which

- (3) A sentence  $S$  is *true* at a context  $c$  if, and only if,  $S$  is true at every point of evaluation  $\langle c, e \rangle$  such that  $e$  belongs to the set  $E(c)$  of circumstances of evaluation determined by  $c$ ;

A sentence  $S$  is *false* at a context  $c$  if, and only if,  $S$  is false at every point of evaluation  $\langle c, e \rangle$  such that  $e$  belongs to the set  $E(c)$  of circumstances of evaluation determined by  $c$ ;

otherwise  $S$  is *neither true nor false* at  $c$ ;

where the notion of sentence-truth at a point of evaluation is defined as follows:

- (4) A sentence  $S$  is true at a point of evaluation  $\langle c, e \rangle$  where  $c$  is a context and  $e$  some circumstances of evaluation if, and only if, the proposition  $p$  expressed by  $S$  in  $c$  is true with respect to  $e$

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<sup>1</sup> See Thomason (1970).

(henceforth, I will use ‘SBT-theorists’ to refer to theorists endorsing both the branching picture of time and (3)). SBT-theorists can simply take circumstances of evaluation to be possible worlds and endorse, accordingly, the following instance of (3):

(5) A sentence  $S$  is *true* at a context  $c$  if, and only if,  $S$  is true with respect to every point of evaluation  $\langle c, w \rangle$  such that  $w$  is a world overlapping at  $c$ ;

A sentence  $S$  is *false* at a context  $c$  if, and only if,  $S$  is true with respect to every point of evaluation  $\langle c, w \rangle$  such that  $w$  is a world overlapping at  $c$ ;

otherwise  $S$  is *neither true nor false* at  $c$ ;

Assuming, then, the highly intuitive principle according to which

(6) An utterance  $u$  is true if, and only if, the sentence  $S$  uttered by means of  $u$  is true at the context  $c$  in which  $u$  is uttered

SBT-theorists can, thus, rightly predict that—since only in some worlds overlapping at  $c_1$  it is sunny at  $t_2$ —my utterance of (1) at  $c_1$  is neither true nor false, thereby vindicating the indeterminacy intuition.

Our scenario also appears to elicit a second intuition: the *determinacy intuition* (MacFarlane 2003,2008).<sup>2</sup> Suppose in fact, that we are placed in the sunny context  $c_2$  and that we look back at the utterance of (1) I produced in  $c_1$ . In this case, the following claim appears to be intuitively true:

(7) The utterance  $u$  of (1) at  $c_1$  was true

Given (5) and (6), however, the SBT-theorist cannot but conclude that my utterance was neither true nor false, contrary to what the determinacy intuition mandates.

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<sup>2</sup> See MacFarlane (2003,2008).

I will call the tension between the SBT-theory and the determinacy intuition the *puzzle of retrospective determinacy*.<sup>3</sup>

## 2. Retrospective determinacy and propositions

MacFarlane (2008) has recently advanced a promising strategy for the solution of the puzzle of retrospective determinacy. It consists of two parts:

I. First, our intuitions do not seem, at a closer scrutiny, to be about the highly technical notion of ‘utterance-truth’. Instead, they appear to be about ‘what is said’ by our utterances and assertions: a *proposition*. Therefore, (7) should be replaced with:

(8) *What I said yesterday* was true

II. Second, in statements like (8), ‘true’ occurs as a monadic predicate for propositions. Its semantics appear to be as simple and straightforward as:

(9) ‘true’ applies to  $x$  at a point of evaluation  $\langle c, w \rangle$  if, and only if,  $x$  is a proposition and  $x$  is true at  $w$

Notice that (9) has two immediate consequences: (i) the absence of an argument place for a time in ‘true’ deprives its tensed uses of any semantic significance (so that the use of ‘was true’ instead of ‘is true’ is determined by grammatical reasons only); (ii) the following disquotational schema is valid:

(10)  $\forall x((x = \text{the proposition that } S) \supset (\text{true}(x) \equiv S))$ <sup>4</sup>

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<sup>3</sup> See MacFarlane (2003,2008).

<sup>4</sup> See MacFarlane (2008).

Given I. and II., the following argument can be used to prove, for instance, that what I said yesterday by uttering ‘It will be sunny at  $t_2$ ’ was true:

Argument A

(A1) Yesterday I uttered the sentence ‘It will be sunny at  $t_2$ ’ [premiss]

(A2) Yesterday I said that it would be sunny at  $t_2$  [from (A1)]

(A3) It is sunny at  $t_2$  [premiss]

(A4) What I said yesterday was true [from (A2),(A3),(10)]

In other words: by uttering ‘It will be sunny at  $t_2$ ’ in  $c_1$ , I expressed the proposition  $P$  that it is sunny at  $t_2$ , which is also the proposition I have just expressed today (in  $c_2$ ) by uttering ‘It is sunny at  $t_2$ ’. Therefore, since  $P$  is true and tensed uses of ‘true’ have no semantic import, it is also true to say today that what I said yesterday *was true*, as the determinacy intuition mandates.

### 3. Adding ‘actually’

Once the puzzle of retrospective determinacy is expressed in terms of propositional truth, SBT-theorists seem to be in position to fully accommodate the determinacy intuition. MacFarlane (2008), however, claims that appearances are deceptive. The problem, he argues, lies in the use of ‘actually’, as in

(11) It will actually be sunny at  $t_2$

As a matter of fact, ‘actually’ appears to be constrained by the following principle of *Initial Redundancy*:

- (12) An operator  $\bullet$  is *initial-redundant* just in case for all sentences S, ' $\bullet$ S' is true at exactly the same contexts of utterance as S (equivalently: each is a logical consequence of the other).

The semantics for 'actually' in standard (non-branching) frameworks,

- (13) 'actually:S' is true at the point of evaluation  $\langle c, w \rangle$  if, and only if, S is true at  $\langle c, w_c \rangle$ , where  $w_c$  is the world of the context  $c$ ,<sup>5</sup>

respect Initial Redundancy because the actuality operator shifts the world of evaluation to the world of the context of utterance 'no matter how far the world of evaluation has been shifted' (MacFarlane 2008:98). In a branching framework, however, (13) will not do, since the openness of the future entails that there is no such thing as *the* world of the context of utterance. MacFarlane proposes, thus, the following definition for the actuality operator in a SBT-setting:

- (14) 'actually:S' is true at  $\langle c, w \rangle$  if, and only if, S is true at every point of evaluation  $\langle c, w' \rangle$ , where  $w'$  is a world overlapping at  $c$ .

According to (14), the actuality-operator behaves as a universal quantifier over the set of worlds overlapping at the context of utterance, thus respecting Initial Redundancy.

Suppose, then, that yesterday (in  $c_1$ ) I uttered (11) and that today we are located in the sunny context  $c_2$ . It seems that, since—by (14)—'actually' quantifies over the set of worlds overlapping at  $c_1$ , the truth of the proposition I expressed yesterday depends on whether or not it is sunny at  $t_2$  *in all the worlds overlapping at  $c_1$* . Since, however, this is not the case, SBT-theorists cannot but predict that it is *false* for me to say today that what I said yesterday (by uttering (11)) was true.

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<sup>5</sup> See Kaplan (1989: 545).

#### 4. 'actually' as indexical

Consider the following argument:

##### Argument B

- (B1) Yesterday I uttered the sentence 'It will actually be sunny at  $t_2$ ' [premiss]
- (B2) Yesterday I said that it would be actually sunny at  $t_2$  [from (B1)]
- (B3) It is actually sunny at  $t_2$  [premiss]
- (B4) What I said yesterday was true [from (B2),(B3) by (10)]

Suppose that (B1) and (B3) are true. If the argument was valid within the SBT-theory, it would follow that (B4) is true and, therefore, that SBT-theorists could—contrary to what MacFarlane claims—account for the determinacy intuition also when 'actually' is concerned. Therefore, if MacFarlane is right, B must be invalid within the SBT-theory. However, since MacFarlane is assuming the validity of (10), the only passage he can blame for the alleged invalidity of B is the transition from (B1) to (B2). This, in turn, appears to entail that, according to MacFarlane, if (14) is the correct definition of 'actually', the proposition expressed by 'It will actually be sunny at  $t_2$ ' at  $c_1$  is different from the one expressed by 'It is actually sunny at  $t_2$ ' at  $c_2$ . Consider, in fact, the following argument:

##### Argument C

- (C1) Yesterday (by uttering the sentence 'It will actually be sunny at  $t_2$ ') I expressed the proposition P1 [premiss]
- (C2) Today (by uttering the sentence 'It is actually sunny at  $t_2$ ') I have expressed the proposition P2 [premiss]
- (C3) It is actually sunny today (at  $t_2$ ) [premiss]

(C4) P2 is true [from (C2), (C3) by (9)]

(C5) P1 and P2 are the same proposition [premiss]

(C5) What I said yesterday was true [from (C1),(C4),(C5) by (10)]

Argument C is valid. The only premiss MacFarlane could reject as false is (C5), that is the premiss according to which ‘It will actually be sunny at  $t_2$ ’ at  $c_1$  and ‘It is actually sunny at  $t_2$ ’ at  $c_2$  express the very same proposition. The transition from (B1) to (B2) must, therefore, be invalid for this very reason. Yesterday (by uttering ‘It will actually be sunny at  $t_2$ ’) I expressed P1. P1 is different from P2 (the proposition that ‘It is actually sunny at  $t_2$ ’ expresses today). Hence, I cannot report *today* what I said yesterday by claiming that yesterday I said *that it would actually be sunny at  $t_2$* , because this would be tantamount to claiming that yesterday I expressed P2, contrary to what we are assuming.

In the framework we are considering, only if the semantic value of ‘actually’ can vary from context to context, the sentences ‘It will actually be sunny at  $t_2$ ’ and ‘It is actually sunny at  $t_2$ ’ can express two different propositions in  $c_1$  and  $c_2$ , respectively.<sup>6</sup> It seems, therefore, that C and B are unsound only if ‘actually’ is an *indexical* expression, that is an expression whose semantic value is a function of the context in which it is uttered.

MacFarlane’s implicit argument against the SBT-theory can thus be reconstructed as follows:

#### Argument D

(D1) ‘actually’ is initial-redundant [premiss]

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<sup>6</sup> SBT-theorists take circumstances of evaluation to be only possible worlds and, hence, ‘treat temporal modifiers as referring terms and quantifiers rather than operators’ (MacFarlane, 2008: 82). Therefore, for them, ‘It will be sunny at  $t_2$ ’ and ‘It is sunny at  $t_2$ ’ express the same proposition as uttered in  $c_1$  and  $c_2$ , respectively.



- (D2) The initial redundancy of ‘actually’ is respected only if ‘actually’ is defined as returning—*somehow*—the world of evaluation to the worlds overlapping at the context of utterance [premiss]
- (D3) ‘actually’ is an operator behaving always as an universal quantifier over the worlds overlapping at the context of utterance [from (D2)]
- (D4) (5) is the correct semantics for ‘actually’ [from (D3)]
- (D5) ‘actually’ is indexical [from (D4)]
- (D6) If ‘actually’ is indexical, then argument B is invalid and, hence, SBT-theorists cannot vindicate the determinacy intuition when actuality-sentences are concerned [premiss]
- (D7) SBT-theorists cannot vindicate the determinacy intuitions when actuality-sentences are concerned [from D5,D6]

I will not dispute here either the transition from (D4) to (D5) or the truth of (D1), (D2) and (D6). Furthermore, for the time being, I will also assume that (D3) follows from (D2). The crucial point is, therefore, whether (D4) follows from (D3), that is whether (5) is the only possible semantics for ‘actually’, once we define ‘actually’ as an operator behaving always as an universal quantifier over the worlds overlapping at the context of utterance. The answer, as I shall argue, must be negative.

##### 5. *Context-sensitivity: indexical/nonindexical*

Interestingly enough, it is MacFarlane himself that provides us with the proper theoretical tools to contrast his claims about the puzzle of retrospective determinacy. As a matter of fact, in his ‘Nonindexical Contextualism’ (2009) he disentangles two notions that appear to have always been conflated in the literature on contextualism: *indexicality* and *context-*

*sensitivity*. According to MacFarlane's lesson, there are in general *two* ways in which a certain expression *e* might be sensitive to a certain feature *g* of the context of utterance:

- (i) the *proposition* expressed by a sentence *S* containing *e* depends on the feature *g* of the context;
- (ii) although the proposition expressed by a sentence *S* containing *e* does *not* depend on the feature *g* of the context, (a) *g* is part of the circumstances of evaluation (that is: the *n*-tuple of parameters representing the circumstances of evaluation comprises a *g*-parameter) and (b) the relevant *g* is the *g* of the context (the *g*-parameter is 'initialized' by the context of utterance).

Although in both cases *e* is *context-sensitive*, in the first case, *e* is an *indexical* expression; in the second case, *e* is a *nonindexical* expression.

## 6. 'actually' as nonindexical

It is relatively easy to give a *nonindexical* definition of 'actually' meeting our *desiderata*:

- (i) First, we have to enrich our indices (that is the *n*-tuples  $\langle x, y, z, \dots \rangle$  representing the circumstances of evaluation) with a *set-of-worlds* parameter *s*, (the 'actuality parameter') and take, consequently, a point of evaluation to be a  $\langle \text{context, world, set of worlds} \rangle$  triple.
- (ii) Second, we define the actuality operator as follows:
  - (15) 'actually:*S*' is true at a point of evaluation  $\langle c, w, s \rangle$  (where *c* is a context, *w* is a world and *s* is a set of worlds) if, and only if, *S* is true at every point of evaluation  $\langle c, w', s \rangle$ , where *w'* is a world belonging to *s*.

(iii) Finally, we substitute the definition of sentence-truth at a context given in (5) with

(16) S is true at a context  $c$  if, and only if, S is true at every point of evaluation  $\langle c, w, s_c \rangle$ , such that  $w$  is a world overlapping at  $c$  and  $s_c$  is the set of worlds overlapping at  $c$ ;

S is false at the context  $c$  if, and only if, S is false at every point of evaluation  $\langle c, w, s_c \rangle$ , such that  $w$  is a world overlapping at  $c$  and  $s_c$  is the set of worlds overlapping at  $c$ ;

otherwise, S is neither true nor false at  $c$ .

Within this theory, three facts assure that ‘actually’ always behaves as a universal quantifier over the set of worlds overlapping at the context of utterance, thus respecting (D3) and Initial Redundancy:

(a) by (15), the truth-conditions for ‘actually’ involve a universal quantification over the set of worlds represented by the actuality parameter;

(b) by (16), the actuality parameter is initialized by the context of utterance as the set of worlds overlapping at the context of utterance;

(c) in the absence of an operator capable of shifting the actuality-parameter,<sup>7</sup> once it has been initialized by the context of utterance, it retains the same value ‘no matter how far the world of evaluation has been shifted’.

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<sup>7</sup> Stanley (2005) has recently attacked—drawing on Lewis (1980)—the position according to which some elements of the circumstances cannot be shifted by any sentence operator. For a plausible defence from this objection see MacFarlane (2009: 245).

The actuality operator so defined is clearly nonindexical. As a matter of fact, although it is indeed sensitive to the set of worlds overlapping at the context of utterance, such a feature of the context is simply part of the circumstances of evaluation and does not affect the proposition expressed by an actuality-sentence. However, if ‘actually’ is nonindexical, arguments B and C are sound. The nonindexicality of ‘actually’ entails, in fact, that the proposition expressed by ‘It will be actually be sunny at t2’ in  $c_1$  is the very same proposition that ‘It is actually sunny at t2’ expresses in  $c_2$ . Therefore, B and C are sound arguments. However, if B and C are sound, then SBT-theorists have no trouble at all with the determinacy intuition, contrary to what MacFarlane claims.

### 7. Is ‘actually’ nonindexical?

So far, I have argued that, if ‘actually’ is thought of as a nonindexical expression, then the puzzle of retrospective determinacy poses no threat to the SBT-theory. In this section, I will advance a simple consideration to bolster this idea.

Within a branching framework, if I utter today

(17) There will be a sea battle tomorrow

my utterance is neither true nor false because there is no possible future marked as ‘special’, among all the futures branching from the present context. The SBT-theory rightly predicts so. Suppose, however, that today I utter

(18) There will *actually* be a sea battle tomorrow

According to the semantics given by MacFarlane ((14) above), my utterance is *false*, since it is false that in *every* possible future there is a sea battle tomorrow. Moreover, given (5), all the sentences of the form ‘actually:S’ are *bivalent* (with respect to any context  $c$ ). However, the reason why (17) is deemed to be gappy in a branching setting is that no possible future is marked as *the* way things will turn out to be. In turn, this seems

to be equivalent to saying that no future is marked as the way things will *actually* turn out to be. Therefore, it seems that, within a branching setting, not only (18) should get the same truth value as (17), but also that, in general, sentences of the form ‘actually:S’ should be neither true nor false at all (and only) the contexts in which S is neither true nor false. In other words, it appears that ‘actually’ should be constrained not only by the principle of Initial Redundancy, but also by the principle—that we might dub *Initial Equivalence*—according to which:

- (19) An operator  $\bullet$  is *initial-equivalent* just in case for all sentences S and contexts  $c$ , S is true/false/neither at a context  $c$  if, and only if, ‘ $\bullet$ S’ is true/false/neither at  $c$ .

There is a simple *nonindexical* definition of ‘actually’ that can be added to the SBT-theory to meet Initial Equivalence and make (18) neither true nor false. It is sufficient to reformulate (15) and (16) above taking the actuality-parameter to be simply a possible world (instead of a set of worlds), as follows:

- (20) ‘actually:S’ is true at  $\langle c, w, w' \rangle$  (where  $c$  is a context and  $w$  [the world of evaluation] and  $w'$  [the actuality-parameter] are possible worlds) if, and only if, S is true at the point of evaluation  $\langle c, w', w' \rangle$
- (21) S is true at a context  $c$  if, and only if, S is true at every point of evaluation  $\langle c, w, w \rangle$ , such that  $w$  is a world overlapping at  $c$ ;
- S is false at a context  $c$  if, and only if, S is false at every point of evaluation  $\langle c, w, w \rangle$ , such that  $w$  is a world overlapping at  $c$ ;
- otherwise, S is neither true nor false at  $c$ .

It is straightforward to see that it is a consequence of (20) and (21) that:

- (22) S is true/false/neither at a context  $c$  if, and only if, ‘actually:S’ is true/false/neither at  $c$ .

and that, therefore, ‘actually’ is not only initial-redundant but also initial-equivalent.

As the actuality operator defined in (15), also the operator defined in (20) is clearly nonindexical. It seems, on the other hand, that no indexical account of ‘actually’ can be added to the SBT-theory to comply with Initial Equivalence. The argument goes as follows:

Argument E

- (E1) ‘actually’ is (somehow) sensitive to the world(/s) overlapping at the context of utterance [premiss]
- (E2) Within a SBT-theory, ‘actually’ is to be defined by means of the notion of truth-at-a-point-of-evaluation [from the definition of SBT-theory]
- (E3) The notion of truth-at-a-point-of-evaluation is bivalent [from the definition of SBT-theory]
- (E4) What only a *context* can provide for the semantics of ‘actually’ in terms of possible worlds within a SBT-setting is a *set* of worlds (that is, the set of worlds overlapping at the context) [from the definition of branching time]
- (E5) If ‘actually’ is indexical, the truth value of ‘actually:S’ at a certain point of evaluation  $\langle c, w \rangle$  depends on the context parameter  $c$  [from the definition of indexicality]
- (E6) If ‘actually’ is indexical, the truth value of ‘actually:S’ at a certain point of evaluation  $\langle c, w \rangle$  depends on the set of worlds overlapping at  $c$  [from E4, E5]

- (E7) If ‘actually’ is indexical, then ‘actually:S’ is true at a point of evaluation  $\langle c, w \rangle$  if the set of worlds overlapping at  $c$  is *such-and-such*; otherwise ‘actually:S’ is false at  $\langle c, w \rangle$  [from E1, E2, E3, E6]
- (E8) If ‘actually’ is indexical, then ‘actually:S’ is true at a context  $c$  if the set of worlds overlapping at  $c$  is *such-and-such*; otherwise ‘actually:S’ is false at  $c$  [from E7, (2)]
- (E9) If ‘actually’ is indexical, then for every sentence  $S$  and context  $c$ , ‘actually:S’ is either true or false at  $c$  [from E8]
- (E10) For some sentence  $S$  and context  $c$ ,  $S$  is neither true nor false at  $c$  [premiss]
- (E11) ‘actually’ is initial-equivalent only if, for some sentence  $S$  and context  $c$ , ‘actually:S’ is neither true nor false at  $c$  [from the definition of Initial Equivalence, E10]
- (E12) If ‘actually’ is indexical, it is not initial-equivalent [from E11]

E is valid; (E1) is intuitively true; (E2)-(E4) cannot be denied without giving up either branching time or supervaluationism; (E5) appears to encapsulate the very gist of the notion of indexicality; denying (E10) would be equivalent to rejecting the very idea that the future is open. Therefore, we have to conclude that, if SBT-theorists accept Initial Equivalence as a constraint on ‘actually’, they are committed to a nonindexical account of ‘actually’ as, for instance, the one given in (20).

I conclude, therefore, that, contrary to what MacFarlane claims, the puzzle of retrospective determinacy poses no serious threat to the supervaluationist treatment of branching time.<sup>8</sup>

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<sup>8</sup> *Acknowledgments*

## References

- Kaplan, D. 1989. Demonstratives. In: J. Almog, J. Perry, and H. Wettstein (eds.): *Themes from Kaplan*. Oxford University Press, 483–563.
- Lewis, D. 1980. Index, Context and Content. In: S. Kanger and S. Øhman (eds.): *Philosophy and Grammar*. Dordrecht: Reidel, 79–100.
- MacFarlane, J. 2003. Future Contingents and Relative Truth. *The Philosophical Quarterly*, 53(212): 321–336.
- MacFarlane, J. 2008. Truth in the Garden of Forking Paths. In: M. Kölbel and M. García-Carpintero (eds.): *Relative Truth*. Oxford University Press, 81–102.
- MacFarlane, J. 2009. Nonindexical Contextualism. *Synthese*, 166(2), 231–250
- Stanley, J. 2005. *Knowledge and Practical Interests*. Oxford: Oxford University Press.
- Thomason, R. H. 1970. Indeterminist Time and Truth value Gaps. *Theoria*, 36: 264–81.